Let P be the point $(-1, -5, 4)$.
Let Q be the point $(1, -4, 3)$.
Let R be the point such that $\overrightarrow{PR} = -\vec{i} + \vec{k}$.

1 POINT EACH
EXCEPT AS NOTED

[a] Find the co-ordinates of R.

$$< x-(-1),\ y-(-5),\ z-4 > = < -1, 0, 1 >$$

$$x+1=-1 \qquad y+5=0 \qquad z-4=1$$

$$x=-2 \qquad y=-5 \qquad z=5$$

$$(-2, -5, 5)$$

[b] Find the area of triangle PQR.

3

$$\overrightarrow{PQ} = < 1-(-1), -4-(-5), 3-4 > = < 2, 1, -1 >$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = < 2, 1, -1 > \times < -1, 0, 1 > = < 1(1)-(-1)0, (-1)(-1)-2(1), 2(0)-1(-1) > = < 1, -1, 1 >$$

$$\tfrac{1}{2} \| \overrightarrow{PQ} \times \overrightarrow{PR} \| = \tfrac{1}{2}\sqrt{1^2 + (-1)^2 + 1^2} = \tfrac{\sqrt{3}}{2}$$

OK IF YOU FOUND $\overrightarrow{PR} \times \overrightarrow{PQ} = \langle -1, 1, -1 \rangle$

[c] Write $\overrightarrow{PQ}$ as the sum of a vector parallel to $\overrightarrow{PR}$ and a vector perpendicular to $\overrightarrow{PR}$.

$$PROJ_{\overrightarrow{PR}}\overrightarrow{PQ} = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\overrightarrow{PR}\cdot\overrightarrow{PR}}\overrightarrow{PR} = \frac{2(-1)+1(0)-1(1)}{(-1)(-1)+1(1)} < -1, 0, 1 > = \frac{-3}{2} < -1, 0, 1 > = < \frac{3}{2}, 0, -\frac{3}{2} >$$

$$< 2, 1, -1 > - < \frac{3}{2}, 0, -\frac{3}{2} > = < \frac{1}{2}, 1, \frac{1}{2} >$$

$$< 2, 1, -1 > = < \frac{3}{2}, 0, -\frac{3}{2} > + < \frac{1}{2}, 1, \frac{1}{2} >$$

[d] Find an equation for the plane that passes through P, Q and R.

2

Normal vector $= \overrightarrow{PQ} \times \overrightarrow{PR} = <1, -1, 1>$

$$1(x-(-1)) - 1(y-(-5)) + 1(z-4) = 0 \quad \rightarrow \quad (x+1) - (y+5) + (z-4) = 0$$

OR $$1(x-1) - 1(y-(-4)) + 1(z-3) = 0 \quad \rightarrow \quad (x-1) - (y+4) + (z-3) = 0$$

OR $$1(x-(-2)) - 1(y-(-5)) + 1(z-5) = 0 \quad \rightarrow \quad (x+2) - (y+5) + (z-5) = 0$$

OR $$x - y + z = 8$$

ANY OF THESE ARE OK

[e] Find symmetric equations for the line that passes through Q and is parallel to the line $\begin{array}{l} x = 3t - 5 \\ y = t + 4 \\ z = 6 - 2t \end{array}$.

Direction vector $= <3, 1, -2>$

2

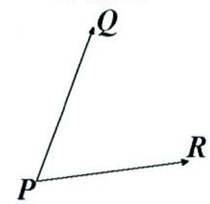
$$\frac{x-1}{3} = \frac{y-(-4)}{1} = \frac{z-3}{-2} \rightarrow \frac{x-1}{3} = y + 4 = \frac{3-z}{2}$$

2

[f] Find the angle $\angle QPR$.

$\angle QPR$ is the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$

NOT DRAWN TO SCALE



$$\cos^{-1} \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\| \overrightarrow{PQ} \| \| \overrightarrow{PR} \|} = \cos^{-1} \frac{-3}{\sqrt{2^2 + 1^2 + (-1)^2}\sqrt{(-1)^2 + 1^2}}$$

$$= \cos^{-1} \frac{-3}{\sqrt{6}\sqrt{2}} = \cos^{-1} \frac{-3}{\sqrt{12}} = \cos^{-1} \frac{-3}{2\sqrt{3}} = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 150^\circ \text{ OR } \frac{5\pi}{6}$$

[g] Find a vector of magnitude 3 that is perpendicular to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

$\overrightarrow{PQ} \times \overrightarrow{PR} = <1, -1, 1>$ is perpendicular to both vectors

$$3 \frac{1}{\| \sqrt{1^2 + (-1)^2 + 1^2} \|} <1, -1, 1> = \frac{3}{\sqrt{3}} <1, -1, 1> = \sqrt{3} <1, -1, 1> = <\sqrt{3}, -\sqrt{3}, \sqrt{3}>$$

[h] If $\overrightarrow{PR}$ represents a force applied to an object as it moves from P to Q, find the work done.

$$\overrightarrow{PR} \cdot \overrightarrow{PQ} = -3$$